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Higher codimension braneworlds from intersecting branes

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Abstract: We study the matching conditions of intersecting brane worlds in Lovelock gravity in arbitrary dimension. We show that intersecting various codimension 1 and/or codimension 2 branes one can find solutions that represent energy-momentum densities localized in the intersection, providing thus the first examples of infinitesimally thin higher codimension braneworlds that are free of singularities and where the backreaction of the brane in the background is fully taken into account.

Keywords: Field Theories in Higher Dimensions, p-branes, Classical Theories of Gravity.

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1. Introduction

Extended objects of arbitrary dimension, the so-called branes, are nowadays a common ingredient of beyond the Standard Model theories that hypothesise the existence of extra dimensions in our universe. These objects can be described in terms of topological defects of field theories in higher dimensions or can have a more fundamental character like the D-branes of string theory. Phenomenological models containing branes have proven to be useful in attacking many of the main problems of high energy physics theories. The important property that is exploited in order to construct these models is that some of the fields can be confined to the submanifold of spacetime regarded as a brane: the zero modes of the topological defects or the gauge theories living in the worldvolume of the string theory D-branes. At low energies, from the point of view of gravity the brane will yield then a distributional term in the energy-momentum tensor that has a delta-like behaviour, *i.e.* an energy-momentum density localized in some submanifold of the whole spacetime. It is then of great importance for these models to find solutions of higher dimensional gravity that correspond to these configurations. The nature of the solutions found depend crucially on the codimension of the brane, *i.e.* the number of extra dimensions. In fact, in Einstein gravity, one can establish an analogy between the behaviour of a codimension n brane and solutions corresponding to a point particle in $n+1$ dimensions. Gravity is trivial in 1+1 dimensions (the action is a topological invariant) while in 2+1 a point particle simply produces a conical deficit at its position [1], the spacetime being flat where there is no matter in both cases. Analogously, the codimension 1 brane simply produces a jump in the

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first derivatives of some metric components at its position and can be dealt with using the Israel junction conditions [2, 3], while a codimension 2 brane produces a conical singularity in the transverse space [4].[1]

Actually, the situation is more subtle for codimension 2 branes. One can find well behaved solutions for a pure tension brane in Einstein gravity (when the brane energy-momentum tensor is proportional to the induced metric) just including a deficit angle in the spacetime, but for a general brane it seemed impossible to find solutions if one requires a non-singular induced metric on the brane [6]. This situation was anticipated in [7], where general solutions with distributional sources were studied in 4D Einstein gravity and it was found that only matter shells (codimension 1 sources) gave a well behaved solution. A way out of this problem was proposed in [8], where it was shown that if one includes a Gauss-Bonnet term in the action[2] the matching conditions can be satisfied for a general brane energy-momentum tensor and, moreover, requiring regularity of the solution one recovers the lower dimensional Einstein equations for the induced metric and matter on the brane from the matching conditions, independently of the bulk solution.

For $n > 2$ the situation is quite different, and one finds singularities (black holes) in the metric describing point particles in $n+1$ dimensions (see [7] for a discussion in 4 dimensions). In the same fashion, known solutions of higher codimension branes present singularities at its positions, naked [11] or surrounded by an event horizon (the black branes of [12]). This is the main reason why higher codimension braneworlds have not been used in the literature to construct phenomenological models as much as their lower codimension cousins, since one does not have a well defined submanifold at the position of the brane. In fact, the usual procedure when dealing with higher codimension branes is to neglect the effect of the brane on the background. However, it is then hard to make any prediction about the nature of the gravity induced on the brane (*i.e.* how gravity for brane observers is), since what one has to compute is the effect of matter on the brane on its own induced metric. In any case, if the curvature grows as one approaches the brane, the solution is singular at its position and the singularity is cut-off by a finite brane width, physical predictions would depend on the brane width and internal structure. It is therefore interesting to look for solutions that are not singular at the position of the brane since in this case one can make unambiguous predictions valid for any microscopic brane theory. For the codimension 2 case, this can be done when one considers the Gauss-Bonnet term in the action [8], and it was found that the nature of gravity on the brane does not yield the result expected from the arguments exhibited in [13], where the self-gravity of the brane is neglected.

[1]This property makes 6D models where observable fields are confined to a 3-brane attractive from the point of view of the Cosmological Constant Problem, since selftuning ideas can find in them a natural implementation [4, 5].

[2]In every odd number of dimensions, 2N+1, one can add to the action a term of Nth order in the curvature tensor (the Nth order Euler density) and the equations of motion remain second order differential equations for the metric. These lagrangians are known as the Lanczos-Lovelock lagrangians [9] and are believed to arise as the low energy limit of string theory, since they are the only ghost-free effective actions for spin two fields [10].

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On the other hand, it has been shown that if one considers the intersection of two codimension 1 branes in 6D Einstein-Gauss-Bonnet gravity one needs the presence of a codimension 2 brane with non-zero tension at the intersection in order to satisfy the maching conditions, due to the contribution of the Gauss-Bonnet terms [14]. This is an interesting observation because the origin of the $\delta^{(2)}$ contribution in the Einstein-Gauss-Bonnet tensor is completely different from a deficit angle, and provides thus another way to find solutions that represent non-singular codimension 2 branes. In this paper we generalize these ideas considering the intersection of various codimension 1 and/or 2 branes. We show that when one considers the most general theory of gravity in higher dimensions[3] (the lagrangian will have the Einstein-Hilbert term plus the dimensionally continued Euler densities), one can find non-singular solutions representing branes of higher codimension living in the intersection of higher dimensional branes (of lower codimension). Branes of codimension up to $N-2$ ($N-1$) in N even (odd) dimensions can be reproduced in this way with a non-singular brane induced metric. Intersecting branes are interesting from the point of view of string phenomenology since one can build up models with a Standard Model like spectrum (see [15] for a recent review). Our results indicate that they are also interesting in the sense that one can build non-singular solutions corresponding to these configurations where the backreaction of the brane in the background is fully taken into account even when higher codimension branes are present.

2. Brane intersections and matching conditions in Lovelock gravity

The Lovelock lagrangian in D dimensions is built up with all the Euler densities of lower dimensions

$$\mathcal{L}_D = \sum_{p=0}^{[\frac{D-1}{2}]} \alpha_p \mathcal{L}_{(p)}\,, \tag{2.1}$$

where α_p is a coefficient of mass dimension $D-2p$ and the square brackets represent the integer part since higher Lovelock terms are trivial in D dimensions. In particular α_0 and α_1 represent just a cosmological constant and the higher dimensional Planck mass, respectively (we will set $\alpha_1 = 1$ in the following without loss of generality). The order p term is

$$\mathcal{L}_{(p)} = \frac{1}{2^p}\delta^{i_1\ldots i_{2p}}_{j_1\ldots j_{2p}} R^{j_1 j_2}{}_{i_1 i_2}\cdots R^{j_{2p-1}j_{2p}}{}_{i_{2p-1}i_{2p}}, \tag{2.2}$$

where $\delta^{i_1\ldots i_{2p}}_{j_1\ldots j_{2p}}$ is the Kronecker symbol of order $2p$ and R^A_{BCD} is the $D-$dimensional Riemann tensor.

The resulting equations of motion are

$$\sum_{p=0}^{[\frac{D-1}{2}]} \alpha_p G_{(p)AB} = T_{AB}\,, \tag{2.3}$$

[3]The only requirement is that the action is torsion free and the equations of motion are second order differential equations for the metric [10].

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where T_{AB} is the energy-momentum tensor and

$$G_{(p)\ B}^{\ A} = -\frac{1}{2^{p+1}}\delta^{Ai_1\dots i_{2p}}_{Bj_1\dots j_{2p}} R^{j_1 j_2}{}_{i_1 i_2}\cdots R^{j_{2p-1}j_{2p}}{}_{i_{2p-1}i_{2p}}. \tag{2.4}$$

We have seen that branes with codimension 1 or 2 admit regular solutions in Einstein gravity[4] whereas branes of codimension 3 or higher do not. The reason is that the Riemann tensor of a general non-singular metric can only have uni- and bi-dimensional delta-like behaviour through discontinuities of the first derivatives of the metric but not higher dimensional ones. A natural way of obtaining higher codimension branes is then by considering two or more branes that intersect and higher orders in the curvature expansion in such a way that the product of Riemann tensors (or contractions thereof) gives the product of delta functions at the intersection of the branes. The use of the Lanczos-Lovelock lagrangian ensures that unaccountable for singularities of the type $\delta(y)^2$ are absent in the higher curvature corrections. In this section we will illustrate this mechanism for the generation of higher dimensional delta functions in the Einstein-Lanczos-Lovelock tensor, eq. (2.3), with the simplest examples, namely the intersection of two codimension 1 branes, a codimension 1 with a codimension 2 brane and two codimension 2 branes, giving rise to, respectively, codimension 2, 3 and 4 branes living in the intersection. We will then generalise this construction to the intersection of an arbitrary number of codimension 1 or 2 branes. In section 3 we give two explicit examples of solutions, first a codimension 3 brane arising at the intersection of a codimension 1 brane with a codimension 2 brane in an $AdS_5 \times S^2$ background in Gauss-Bonnet gravity and second a string motivated 10D solution where 5-branes and 3-branes are obtained at the intersections of, respectively, two and three codimension 2 branes in a (Minkowski)$_4 \times S^6$ background.

2.1 Intersection of codimension 1 branes

In this section we review the example of the intersection of two codimension 1 branes, that has been recently used as a way to obtain a codimension 2 brane world living in the intersection [14]. The backreaction on the background for a codimension 1 brane can be dealt with assuming discontinuities in the first derivatives of the metric with respect to the orthogonal coordinate at the position of the brane, that produces a jump in the extrinsic curvature as one goes from one side of the brane to the other [2]. In the intersection of two such branes a two-dimensional delta function is generated by the second Lovelock (Gauss-Bonnet) term as follows. We consider six-dimensional space-time and take following ansatz for the metric,

$$\mathrm{d}s^2 = g_{\mu\nu}(x,y,z)\,\mathrm{d}x^\mu \mathrm{d}x^\nu - W^2(x,y,z)\,\mathrm{d}y^2 - L^2(x,y,z)\,\mathrm{d}z^2\,. \tag{2.5}$$

We also consider a Z_2 symmetry for each of the "extra" dimensions in $z=0$ and $y=0$, where we locate two codimension 1 branes spanning, respectively, the (x^μ, y) and (x^μ, z)

[4]We consider pure tension branes now for the case of codimension 2, the subtleties associated with a general codimension 2 energy-momentum tensor will be discussed below.

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coordinates. The relevant components of the Riemann tensor for the matching conditions (those with a delta like behaviour) are

$$R^{y\mu}{}_{y\nu} = \frac{1}{2}\frac{g^{\mu\rho}\ddot{g}_{\rho\nu}}{W^2} + \cdots \,, \tag{2.6}$$

$$R^{z\mu}{}_{z\nu} = \frac{1}{2}\frac{g^{\mu\rho}g''_{\rho\nu}}{L^2} + \cdots \,, \tag{2.7}$$

$$R^{yz}{}_{yz} = \frac{\ddot{L}}{W^2 L} + \frac{W''}{L^2 W} + \cdots \,, \tag{2.8}$$

where a dot and a prime denote, respectively, derivatives with respect to y and z and we have used the fact that for a discontinuous first derivative we have

$$\ddot{g}_{\mu\nu}(x,y,z) = 2\dot{g}_{\mu\nu}(x,0^+,z)\ \delta(y) + \cdots \,, \tag{2.9}$$

and similarly with the other components. Using the general expression for the different terms in the Lovelock tensor, eq. (2.3), we obtain the following singular terms in the Einstein tensor (Lovelock term of order 1)

$$\begin{aligned} G^{(1)}_{\mu\nu} &= \frac{1}{W^2}\left[-g_{\mu\nu}\left(2\frac{\dot{L}}{L} + g^{\rho\sigma}\dot{g}_{\rho\sigma}\right) + \dot{g}_{\mu\nu}\right]\delta(y) + \\ &\quad + \frac{1}{L^2}\left[-g_{\mu\nu}\left(2\frac{W'}{W} + g^{\rho\sigma}g'_{\rho\sigma}\right) + g'_{\mu\nu}\right]\delta(z) + \cdots \,, \end{aligned} \tag{2.10}$$

$$G^{(1)}_{yy} = \frac{W^2}{L^2}g^{\rho\sigma}g'_{\rho\sigma}\delta(z) + \cdots \,, \tag{2.11}$$

$$G^{(1)}_{zz} = \frac{L^2}{W^2}g^{\rho\sigma}\dot{g}_{\rho\sigma}\delta(y) + \cdots \,, \tag{2.12}$$

where the dots represent terms without delta functions. As expected there is a term proportional to $\delta(y)$ in the $(\mu\nu)$ and (zz) components of the Riemann tensor and a term proportional to $\delta(z)$ in the $(\mu\nu)$ and (yy) ones, and therefore one can find solutions that represent two codimension 1 branes that intersect at the points $y = z = 0$. In this intersection, when including the second Lovelock (Gauss-Bonnet) term we have to consider the presence of a codimension 2 brane with non-zero tension. This can de seen by computing the coefficient of $\delta(y)\delta(z)$ appearing in $G^{(2)}_{MN}$,

$$\begin{aligned} G^{(2)}_{\mu\nu} &= -\frac{4}{W^2L^2}\Big[g_{\mu\nu}\left(g^{\rho\sigma}\dot{g}_{\rho\sigma}g^{\lambda\tau}g'_{\lambda\tau} - g^{\lambda\sigma}\dot{g}_{\rho\tau}g^{\tau\rho}g'_{\sigma\lambda}\right) - \dot{g}_{\mu\nu}g^{\rho\sigma}g'_{\rho\sigma} - \\ &\quad - g'_{\mu\nu}g^{\rho\sigma}\dot{g}_{\rho\sigma} + \dot{g}_{\mu\rho}g^{\rho\sigma}g'_{\sigma\nu} + g'_{\mu\rho}g^{\rho\sigma}\dot{g}_{\sigma\nu}\Big]\delta(y)\delta(z) + \cdots \end{aligned} \tag{2.13}$$

Now the dots represent terms without delta functions or with just one delta. Notice that the two-dimensional delta function only appears along the (μ, ν) coordinates, the coordinates of a 3-brane sitting in $z = y = 0$. The terms with just one delta appearing in $G^{(2)}_{MN}$ modify the matching conditions for the codimension 1 branes, eqs. (2.10)–(2.12), but we do not write them explicitly since they are somewhat complicated and do not contribute anything to the discussion (general expressions for the matching conditions of codimension 1 branes in Einstein-Gauss-Bonnet gravity can be found in [16]).

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2.2 Intersection of codimension 1 and codimension 2 branes

In this section we shall describe the intersection of a codimension 1 brane with a codimension 2 one, and we will see that, when including the Lovelock terms, one needs the presence of a non-trivial energy-momentum density localized in the intersection, leading to the first example of a codimension 3 brane with a non-singular induced metric. In order to do that consider a seven-dimensional space time with the following metric

$$\mathrm{d}s^2 = g_{\mu\nu}(x,r,y)\,\mathrm{d}x^\mu \mathrm{d}x^\nu - W^2(x,y)\,\mathrm{d}r^2 - L^2(r,x,y)\,\mathrm{d}\theta^2 - \mathrm{d}y^2\,, \tag{2.14}$$

where as in the previous section we consider a Z_2 symmetry around $y=0$, θ has period 2π and in order for these coordinates to represent a codimension 2 submanifold at $r=0$ we must have $L \simeq \beta r + \mathcal{O}(r^2)$ for small r. Similarly to the case of the codimension 1 brane in which the jump in the normal derivatives generate a one-dimensional delta function, if the slope of the function L is not one in $r=0$ ($\beta \neq 1$), a conical singularity (i.e. a two-dimensional delta function) is generated in the Einstein tensor at that point

$$\frac{\partial_r^2 L}{L} = -(1-\beta)\frac{\delta(r)}{L} + \cdots \tag{2.15}$$

We will take this as the only source of $\delta(r)$ behaviour. The possibility of considering $\partial_r g_{\mu\nu}|_{r=0^+} \neq 0$, so $\partial_r^2 g_{\mu\nu} \sim \delta(r)$ would generate also a two-dimensional delta function in the Gauss-Bonnet term [8], but it would lead to a divergent Ricci tensor as one approaches $r=0$ since

$$R_{\mu\nu} = \frac{1}{2}\frac{L'}{L}\partial_r g_{\mu\nu} + \cdots = \frac{\partial_r g_{\mu\nu}}{2r} + \mathcal{O}(1) \tag{2.16}$$

near the brane, so we will consider only solutions in which $\partial_r g_{\mu\nu}|_{r=0^+} = 0$. The relevant (distributional) components of the Riemann tensor are then

$$R^{y\mu}{}_{y\nu} = \frac{1}{2}\frac{g^{\mu\rho}\ddot{g}_{\rho\nu}}{W^2} + \cdots\,, \tag{2.17}$$

$$R^{yr}{}_{yr} = \frac{\ddot{W}}{W} + \cdots\,, \tag{2.18}$$

$$R^{y\theta}{}_{y\theta} = \frac{\ddot{L}}{L} + \cdots\,, \tag{2.19}$$

$$R^{r\theta}{}_{r\theta} = \frac{\partial_r^2 L}{W^2 L} + \cdots\,, \tag{2.20}$$

where a dot denotes a derivative with respect to y and we have, as before,

$$\ddot{g}_{\mu\nu}(x,r,y) = 2\dot{g}_{\mu\nu}(x,r,0^+)\;\delta(y) + \cdots\,, \tag{2.21}$$

and similarly with the other components. Again it is straightforward to obtain the delta-like components of the Einstein tensor:

$$G^{(1)}_{\mu\nu} = (1-\beta)g_{\mu\nu}\frac{1}{W^2}\frac{\delta(r)}{L} + \left[-g_{\mu\nu}\left(2\frac{\dot{L}}{L} + 2\frac{\dot{W}}{W} + g^{\rho\sigma}\dot{g}_{\rho\sigma}\right) + \dot{g}_{\mu\nu}\right]\delta(y) + \cdots\,, \tag{2.22}$$

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$$G^{(1)}_{yy} = -(1-\beta)\frac{1}{W^2}\frac{\delta(r)}{L} + \cdots\,, \tag{2.23}$$

$$G^{(1)}_{rr} = W^2\left[g^{\rho\sigma}\dot{g}_{\rho\sigma} + 2\frac{\dot{L}}{L}\right]\delta(y) + \cdots, \tag{2.24}$$

$$G^{(1)}_{\theta\theta} = L^2\left[g^{\rho\sigma}\dot{g}_{\rho\sigma} + 2\frac{\dot{W}}{W}\right]\delta(y) + \cdots\,, \tag{2.25}$$

where the dots represent terms without deltas while the three-dimensional delta function appearing in the Gauss-Bonnet term is given by

$$G^{(2)}_{\mu\nu} = (1-\beta)\frac{1}{W^2}[g_{\mu\nu}g^{\rho\sigma}\dot{g}_{\rho\sigma} - \dot{g}_{\mu\nu}]\delta(y)\frac{\delta(r)}{L} + \cdots \tag{2.26}$$

where the dots represent terms without three-dimensional delta functions. Notice however that in these terms that we have not written there are contributions proportional to $\delta(y)$ and $\delta(r)/L$ that will modify the matching conditions obtained from the Einstein part. These corrections can be seen as small for the codimension 1 brane (we do not write them here for the same reasons as in the previous section), but as we have previously explained they are crucial for the codimension 2 brane, since in the Einstein term the delta-like contribution is proportional to the brane induced metric [see eqs. (2.22), (2.23)] so in order to find solutions for a codimension 2 brane that has a general energy-monentum tensor (not just pure tension) on has to consider the contribution of the Gauss-Bonnet term to the matching condition.[5]

Notice that the matching condition for the codimension 3 brane at the intersection inherits the richer structure of its codimension 1 parent. (The matching condition is indeed identical to that of a codimension 1 brane in five dimensions.) It is worth pointing out that in 7 dimensions we have also the third Lovelock term at our disposal. This term would contribute to all the matching conditions, but the structure of the Einstein-Lanczos-Lovelock tensor, eq. (2.3), ensures that terms proportional to $\delta(y)^2$ or $\delta(r)^2$ will not appear. Also, it is easy to see that terms proportional to $\delta(y)$ will only appear along the (μ,ν), (r,r) and (θ,θ) components, those proportional to $\delta(r)/L$ will only appear along the (μ,ν) and (y,y) components, while those proportional to $\delta(y)\delta(r)/L$ will only appear in the (μ,ν) components, just contributing subleading corrections to the matching conditions already obtained.

2.3 Intersection of codimension 2 branes

We now turn to the intersection of two codimension 2 branes and we will see that, when including higher Lovelock terms in the action, one needs a codimension 4 brane living in the intersection in order to satisfy the matching conditions. We consider eight space time dimensions with the following ansatz for the metric,

$$\mathrm{d}s^2 = g_{\mu\nu}(x,r_1,r_2)\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu - \mathrm{d}r_1^2 - L_1^2(r_1)\,\mathrm{d}\theta_1^2 - \mathrm{d}r_2^2 - L_2^2(r_2)\,\mathrm{d}\theta_2^2\,, \tag{2.27}$$

[5] This property, comes from a term in eq. (2.26) proportional to $\hat{G}^{(1)}_{mn}\frac{\delta(r)}{L}$ that we have not explicitely written, where $\hat{G}^{(1)}_{mn}$ represents Einstein tensor for the induced metric on the codimension 2 brane.

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where as usual θ_i have period 2π, $L_i = \beta_i r_i + \mathcal{O}(r_i^2)$, $i = 1, 2$ and, in order to avoid curvature singularities as we approach the branes we consider $\partial_{r_i} g_{\mu\nu}|_{r_i=0^+} = 0$. According to our experience with codimension 2 branes and intersections between branes one can expect that the Einstein tensor will allow us to find solutions for a pure tension codimension 2 brane, including the Gauss-Bonnet term we will be able to satisfy the matching conditions for general codimension 2 brane but only for a pure tension codimension 4 brane at the intersection whereas the third Lovelock term, that is also available in eight dimensions, will give enough freedom to match a general energy-momentum tensor for the codimension 4 brane. This intuition is in fact correct as we show now.

Since we have assumed that $\partial_{r_i} g_{\mu\nu}|_{r_i=0^+} = 0$ the only source of delta functions in the Riemann tensor are the conical singularities, and the relevant components of this tensor are then

$$R^{r_i\theta_i}{}_{r_i\theta_i} = \frac{\partial^2_{r_i} L_i}{L_i}\,, \tag{2.28}$$

where again $i = 1, 2$. The Einstein tensor has the following singular components

$$G^{(1)}_{\mu\nu} = -g_{\mu\nu} \sum_{i=1}^{2} \left[(1-\beta_i)\frac{\delta(r_i)}{L_i}\right] + \cdots\,, \tag{2.29}$$

$$G^{(1)}_{r_i r_i} = (1-\beta_j)\frac{\delta(r_j)}{L_j} + \cdots\,, \qquad j \neq i\,, \tag{2.30}$$

$$G^{(1)}_{\theta_i\theta_i} = (1-\beta_j) L_j^2 \frac{\delta(r_j)}{L_j} + \cdots\,, \qquad j \neq i\,, \tag{2.31}$$

while the Gauss-Bonnet tensor has the following codimension 4 singularity,

$$G^{(2)}_{\mu\nu} = -4 g_{\mu\nu}(1-\beta_1)(1-\beta_2)\frac{\delta(r_1)}{L_1}\frac{\delta(r_2)}{L_2} + \cdots\,, \tag{2.32}$$

plus codimension 2 singularities that we do not explicitely write. These codimension 2 deltas generated in the Gauss-Bonnet term are proportional to the Einstein tensors for the induced metrics on the corresponding 5-branes. (The (r_i, r_i) and (θ_i, θ_i) components will of course have the corresponding contribution proportional to $\delta(r_{j\neq i})/L_j$.) Notice that, as expected, if we do not include the next Lovelock term, we will be able to find solutions only for a pure tension 3-brane, since the $\delta^{(4)}$ term is proportional to the brane induced metric [eq. (2.32)]. In order to be able to satisfy the matching conditions for a general energy-momentum tensor for the 3-brane at the intersection we have to make use of the third Lovelock term that has the following codimension 4 delta function

$$G^{(3)}_{\mu\nu} = 24(1-\beta_1)(1-\beta_2)\hat{G}^{(1)}_{\mu\nu}(g)\frac{\delta(r_1)}{L_1}\frac{\delta(r_2)}{L_2} + \cdots\,, \tag{2.33}$$

where $\hat{G}^{(1)}_{\mu\nu}(g)$ is the Einstein tensor for the induced metric on the 3-brane. As always there are lower codimension delta functions in this tensor not explicitly written that will modify the matching conditions for the codimension 2 branes. In fact is easy to see that these corrections will take the form of the Gauss-Bonnet (second Lovelock term) for the induced metric of the respective 5-branes and as we will see in the next subsection this structure is straightforwardly generalisable to higher (co)dimensions.

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2.4 General intersection of codimension 1 and codimension 2 branes

We have shown how Lanczos-Lovelock gravity allows us to obtain up to codimension 4 delta functions in the generalized Einstein tensor in an otherwise regular background at the intersection of two branes of codimensions 1 or 2. These solutions require then the presence of a higher codimension branes living in these intersections. In this section we are going to generalise these examples by discussing the intersection of an arbitrary number of codimension 1 and 2 branes.

In a D-dimensional space time we have non-trivial Lovelock terms up to order $p_{\max} = [\frac{D-1}{2}]$, where as usual the square brackets denote integer part. According to our previous discussion, the fact that we can get one codimension 1 or 2 delta functions in the Riemann tensor of a regular metric (and its contractions) implies that up to $p_{\max}$ branes can have a non-trivial intersection. Consider the intersection of m_1 codimension 1 branes and m_2 codimension 2 branes, where $m_1 + m_2 \leq p_{\max}$. The brane at the common intersection has codimension $m_1 + 2m_2$, which can be up to $2p_{\max}$ when all the branes we use are codimension 2. This means that in D dimensions we can in principle match up to $0-$branes or $1-$branes for D odd and even, respectively. We have obtained those numbers by just counting the number of powers of the Riemann tensor we have. From the general form of the Lovelock equations of motion and the examples we have discussed above it is however evident that for an arbitrary D (we have to consider $D \geq 5$ if we want to have any nontrivial intersection although the argument carries on for lower dimensions with just one brane) solutions representing all the lower dimensional branes down to $0-$ or $1-$ branes can indeed be obtained by means of brane intersections.

When we considered the intersection of two codimension 2 branes, we had to consider the contribution of the third Lovelock term to the matching conditions to be able to satisfy them for a general 4-brane energy-momentum tensor. Remarkably, we found the interesting result that the matching condition implies that the induced metric at this intersection satisfies the lower dimensional Einstein equations, analogously to what happens for the codimension 2 brane. In fact it is easy to see that the equations of motion (obtained from the matching conditions) for the induced metric on the world-volume of a codimension 2 brane in Lovelock gravity of order p correspond to the Lovelock equations of order $p-1$. So for a D-dimensional metric of the form

$$\mathrm{d}s^2 = g_{\mu\nu}(x,r)\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu - \mathrm{d}r^2 - L^2(r)\,\mathrm{d}\theta^2\,, \tag{2.34}$$

where as always $0 \leq \theta \leq 2\pi$, $L = \beta r + \mathcal{O}(r^2)$ and $\partial_r g_{\mu\nu}|_{r=0^+} = 0$, the Lovelock tensor of order p will contribute to the matching condition

$$\begin{aligned} G_{(p)}{}^{\mu}{}_{\nu}|_{\mathrm{Cod}\,2} &= -\frac{4p}{p+1}\delta^{\mu i_1\ldots i_{2p-2}}_{\nu j_1\ldots j_{2p-2}} R^{j_1 j_2}{}_{i_1 i_2}\cdots R^{j_{2p-3}j_{2p-2}}{}_{i_{2p-3}i_{2p-2}} R^{r\theta}{}_{r\theta} + \cdots \\ &= -2p(1-\beta)\frac{\delta(r)}{L}\hat{G}_{(p-1)}{}^{\mu}{}_{\nu} + \cdots\,. \end{aligned} \tag{2.35}$$

In the last equality the hat represents the corresponding Lovelock tensor computed with the induced metric on the brane [*i.e.* $g_{\mu\nu}(x,0)$]. Similarly, if we consider the intersection

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of m codimension 2 branes with a metric of the type

$$\mathrm{d}s^2 = g_{\mu\nu}(x, r_i)\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu - \sum_{i=1}^{m}(\mathrm{d}r_i^2 + L_i^2(r_i)\,\mathrm{d}\theta_i^2)\,, \tag{2.36}$$

with $0 \leq \theta_i \leq 2\pi$, $L_i = \beta_i r_i + \mathcal{O}(r_i^2)$ and $\partial_{r_i} g_{\mu\nu}|_{r_i=0^+} = 0$, the contribution of the order p Lovelock term to the matching condition of the intersection (at all $r_i = 0$) reads

$$G_{(p)\ \nu}^{\ \mu}|_{\mathrm{m\,Cod\,2}} = (-1)^m \frac{2^m p!}{(p-m)!} \hat{G}_{(p-m)\ \nu}^{\ \mu} \prod_{i=1}^{m}(1-\beta_i)\frac{\delta(r_i)}{L_i} + \cdots\,, \tag{2.37}$$

where $\hat{G}_{(p-m)\ \nu}^{\ \mu}$ is the $(p-m)$-th order Lovelock tensor for the induced metric. In particular if we restrict ourselves to an even number of dimensions (in order to eventually obtain 3-branes), $D = 2n$, we have at our disposal $n-1$ Lovelock terms. The order $n-2$ Lovelock term allows us to match a pure Einstein 3-brane at the intersection of $n-2$ codimension 2 branes,

$$G_{(n-2)\ \nu}^{\ \mu}|_{\mathrm{(n-2)\,Cod\,2}} = (-2)^{n-3}(n-2)!\delta_\nu^\mu \prod_{i=1}^{n-2}(1-\beta_i)\frac{\delta(r_i)}{L_i} + \cdots\,, \tag{2.38}$$

whereas using the highest, order $n-1$, Lovelock term we can match a general energy-momentum tensor on the 3-brane at the intersection of the $n-2$ codimension 2 branes since

$$G_{(n-1)\ \nu}^{\ \mu}|_{\mathrm{(n-2)\,Cod\,2}} = (-2)^{(n-2)}(n-1)!\hat{G}_{(1)\ \nu}^{\ \mu} \prod_{i=1}^{n-2}(1-\beta_i)\frac{\delta(r_i)}{L_i} + \cdots \tag{2.39}$$

and we obtain the Einstein equation for the induced metric on the brane. Thus, the structure of the matching conditions for the intersection of an arbitrary number of codimension 2 branes in Lovelock gravity has a "russian doll" structure, with the induced metric in the worldvolume of each brane satisfying the Lovelock equations that corresponds with its dimensionality. An example corresponding to $D = 10$ will be worked out in the next section.

Once we have written the general expression for the intersection of an arbitrary number of codimension 2 branes, we can use it to study the intersection of m_1 codimension 1 and m_2 codimension 2 branes in two steps. First consider the intersection of the m_2 codimension 2 branes. The resulting matching conditions for the brane at such intersection in order p Lovelock gravity are, as we have just shown, the order $p - m_2$ Lovelock equations for the induced metric on the brane intersection. We are therefore left with the problem of what the equations of motion are for the metric induced at the intersection of an arbitrary number of codimension 1 branes. The solution to that problem cannot be written in as neat a way as in the case of codimension 2 branes. The reason is that in the codimension 2 case we are assuming that the extrinsic curvature is zero (*i.e.* $\partial_r g_{\mu\nu}|_{r=0^+} = 0$, with $g_{\mu\nu}|_{r=0}$ the brane induced metric) and we are left with some equations for the induced metric that have a closed form, and do not depend on the bulk structure. For the codimension 1 case this cannot be done since the discontinuity in the extrinsic curvature is the only source

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of delta functions, and to obtain the equations that relate the induced metric with the brane energy-momentum tensor one has to write down the Einstein tensor for the induced metric in terms of the extrinsic curvature plus corrections (this can be done using the Gauss-Codazzi formalism), and the equations of motion for the induced metric can not be obtained in a closed form [3].

3. Explicit examples

So far we have only computed matching conditions in Lovelock gravity and we have seen that higher Lovelock terms give enough freedom to match higher codimension branes at brane intersections. This is an important result, being the first example of local solutions for higher codimension branes in a regular background. It still remains the question however of whether such solutions can be consistent globally. In this section we demonstrate with a couple of simple, but phenomenologically relevant examples that global solutions for higher codimension branes can indeed be obtained. In ref. [14] global solutions are found for a codimension 2 brane arising at the intersection of two codimension 1 branes in Einstein-Gauss-Bonnet gravity. Here we will give two examples of global solutions with one or two codimension 3 branes arising at the intersection of a codimension 1 and a codimension 2 brane (in an $AdS_5 \times S^2$ background) and two codimension 6 (codimension 4) branes at the intersection of three (two) codimension 2 branes in a $(\mathrm{Minkowski})_4 \times S^6$ background. The two solutions shown here do not present any horizon and the background is regular everywhere. We have not made any assuption about the sign of the brane tensions. This could be relevant for questions of stability [17] that are beyond the scope of the present study.

3.1 A 7D model

A simple example for a codimension 3 brane arising at the intersection of a codimension 2 with a codimension 1 brane in a regular globally defined background can be obtained in $AdS_5 \times S^2$ with the following metric

$$\mathrm{d}s^2 = \mathrm{e}^{-k|y|}\eta_{\mu\nu}\mathrm{d}x^\mu\,\mathrm{d}x^\nu - \mathrm{d}y^2 - R^2(\mathrm{d}\theta^2 + \beta^2\sin^2\theta\,\mathrm{d}\phi^2)\,, \tag{3.1}$$

where we have imposed a Z_2 symmetry around $y = 0$, θ ranges from 0 to π while ϕ has the standard periodicity of 2π and we allowed for an arbitrary deficit angle $1 - \beta$. It is easy to see that this metric is a solution of Einstein-Gauss-Bonnet equations in the bulk with the following energy-momentum tensor

$$T^{\mathrm{Bulk}}_{MN} = \begin{pmatrix} \mathrm{e}^{-k|y|}\eta_{\mu\nu}\Lambda_1 & & \\ & -\Lambda_1 & \\ & & \kappa_{ij}\Lambda_2 \end{pmatrix}, \tag{3.2}$$

where κ_{ij} is the metric on the sphere and the two constants k and R are related to the ones appearing in the energy-momentum tensor as

$$\Lambda_1 = -\frac{3}{4}k^2(2 + k^2\alpha_2) + \frac{1 + 6k^2\alpha_2}{R^2}\,, \tag{3.3}$$

$$\Lambda_2 = -\frac{5}{4}k^2(2 + 3k^2\alpha_2)\,. \tag{3.4}$$

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Such an inhomogeneous *vev* for the energy-momentum tensor can be obtained through the flux of a 2- or 5-form (Freund-Rubin compactification [18]) or through an anisotropic Casimir effect [19], for instance.

The jump in the extrinsic curvature at $y = 0$ gives rise to a delta function contribution to the Einstein-Gauss-Bonnet tensor that can be interpreted as the backreaction due to a brane located at $y = 0$ with the following energy-momentum tensor

$$T_{MN}^{\mathrm{Cod}\,1} = \begin{pmatrix} 3k(1+k^2\alpha_2)\eta_{\mu\nu}\delta(y) & & \\ & 0 & \\ & & 4k(1+3k^2\alpha_2)k_{ij}\delta(y) \end{pmatrix}. \tag{3.5}$$

Notice that we need an inhomogeneous form for the brane tension. This can be easily generated again by considering the magnetic flux of a U(1) gauge field or the flux of a 4 form localized on the brane. In any case we find that these values have to be fine tuned with respect to the bulk energy-momentum tensor, as in the original Randall-Sundrum model [20].[6] A value of $\beta \neq 1$ can be interpreted as being the backreaction induced by two codimension 2 branes at $\theta = 0, \pi$ (we can get rid of one of these branes by considering a Z_2 orbifolding of the sphere with respect to the equatorial plane) with energy-momentum tensor given by

$$T_{MN}^{\mathrm{Cod}\,2} = \begin{pmatrix} (1-\beta)(1+6k^2\alpha_2)\mathrm{e}^{-k|y|}\eta_{\mu\nu}\frac{\delta(\sin\theta)}{R^2\beta\sin\theta} & & \\ & -(1-\beta)(1+6k^2\alpha_2)\frac{\delta(\sin\theta)}{R^2\beta\sin\theta} & \\ & & 0 \end{pmatrix}. \tag{3.6}$$

Now the brane tension can take any value (since β is a free parameter of the solution) without modifying the brane geometry. This is a reflection of the fact that in codimension 2 the brane tension does not have to be fine tuned with respect to bulk parameters and thus provide an interesting starting point for building selftuning models [4, 5]. Finally, the Gauss-Bonnet term has a $\delta^{(3)}$ contribution that gives the matching of the codimension 3 brane at the intersection with energy-momentum tensor

$$T_{MN}^{\mathrm{Cod}\,3} = \begin{pmatrix} -12\alpha_2 k(1-\beta)\eta_{\mu\nu}\delta(y)\frac{\delta(\sin\theta)}{R^2\beta\sin\theta} & & \\ & 0 & \\ & & 0 \end{pmatrix}. \tag{3.7}$$

The tension of this brane is fixed in terms of the other parameters of the solution.

3.2 A 10D model

We finally want to show an explicit example of the intersection of codimension 2 branes. Instead of describing the simplest case of two codimension 2 branes we consider the string

[6] This is a generic feature of codimension 1 models. Since the bulk geometry determines the jump in the extrinsic curvature one has to fine tune the brane tension with respect to bulk parameters. There have been however attempts to get rid of this fine tuning by coupling the brane to a scalar field that were the origin of the so-called selftuning models [21].

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motivated one of a codimension 6 brane arising at the intersection of three codimension 2 branes in ten-dimensional space time. In particular we take our space time to be (Minkoski)$_4 \times S^6$ with the following metric

$$\mathrm{d}s^2 = \eta_{\mu\nu}\mathrm{d}x^\mu\,\mathrm{d}x^\nu - R^2\Big[\mathrm{d}\theta_1^2 + \beta_1^2\sin^2\theta_1\mathrm{d}\phi_1^2 + \cos^2\theta_1 \times \\ \times\left(\mathrm{d}\theta_2^2 + \beta_2^2\sin^2\theta_2\mathrm{d}\phi_2^2 + \cos^2\theta_2(\mathrm{d}\theta_3^2 + \beta_3^2\sin^2\theta_3\mathrm{d}\phi_3^2)\right)\Big]\,, \tag{3.8}$$

where $0 \leq \theta_{1,2} \leq \pi/2$, $0 \leq \theta_3 \leq \pi$, $0 \leq \phi_i \leq 2\pi$ and the three arbitrary constants β_1, β_2 and β_3 that are allowed by the symmetries of the six-sphere will allow us to match the energy-momentum tensor of three codimension 2 branes located each one at $\sin\theta_i = 0$, for $i = 1, 2, 3$. Even though up to the fourth order Lovelock term is available in ten dimensions, in our case due to the factorizable form of the metric and the fact that the four non-compact dimensions are flat, all the components of the Lovelock term of order four vanish. Therefore we will show that this metric represents a well behaved globally defined solution with three codimension 2 branes intersecting by pairs on three codimension 4 branes and the three of them at two codimension 6 branes in Lovelock gravity.[7] It is indeed easy to see that the bulk equations of motion are satisfied for the following energy-momentum tensor

$$T_{MN}^{\mathrm{Bulk}} = \begin{pmatrix} \eta_{\mu\nu}\Lambda_1 & \\ & \kappa_{ij}\Lambda_2 \end{pmatrix}, \tag{3.9}$$

where κ_{ij} is the metric for the 6-sphere and the constants $\Lambda_{1,2}$ have to be fine-tuned to get four-dimensional flat space, being related to R as

$$\Lambda_1 = \frac{15}{R^2}\left(1 - \frac{12}{R^2}\alpha_2 + \frac{24}{R^4}\alpha_3\right), \tag{3.10}$$

$$\Lambda_2 = \frac{10}{R^2}\left(1 - \frac{6}{R^2}\alpha_2\right). \tag{3.11}$$

The asymmetric energy-momentum tensor can be obtained as before using the *vev* of the corresponding form or through anisotropic Casimir effect.

Now we turn to the branes. A non-trivial value of the deficit angles, $\beta_i \neq 1$, induces a conical singularity at $\sin\theta_i = 0$, matching an energy-momentum for a codimension 2 brane with the following non-vanishing components

$$T_{\mu\nu}^{\mathrm{Cod}\,2}(i) = \eta_{\mu\nu}(1-\beta_i)\left(1 - \frac{24}{R^2}\alpha_2 + \frac{72}{R^4}\alpha_3\right)\frac{\delta(\sin\theta_i)}{\sqrt{\kappa_i}}\,, \tag{3.12}$$

$$T_{kl}^{\mathrm{Cod}\,2}(i) = \kappa_{kl}(1-\beta_i)\left(1 - \frac{12}{R^2}\alpha_2\right)\frac{\delta(\sin\theta_i)}{\sqrt{\kappa_i}}\,, \qquad k,l \neq \theta_i, \phi_i\,, \tag{3.13}$$

where $i = 1, 2, 3$ and κ_i is the determinant of the 2×2 submatrix of the sphere metric corresponding to the coordinates θ_i, ϕ_i. Note that, according to our general discussion, the

[7] The submanifold defined by the condition $\sin\theta_i = 0$ is nothing but (Minkowski)$_4 \times S^4$ (notice that $\theta_3 = 0$ covers one hemisphere and we need $\theta_3 = \pi$ to cover the full S^4) while one can check that imposing the conditions $\sin\theta_2 = \sin\theta_3 = 0$ we are left with a submanifold that corresponds to (Minkowski)$_4 \times S^2$ (and the same for every other pair of theta angles). Finally, the condition $\sin\theta_1 = \sin\theta_2 = \sin\theta_3 = 0$ results on two 3-branes [(Minkowski)$_4 \times S^0$].

contribution of each Lovelock term to the matching condition for one codimension 2 brane is proportional to the previous Lovelock term for the induced metric on the brane. The fact that we are forcing the worldvolume of the brane to be (Minkowski)$_4 \times S^4$ implies that the brane energy-momentum tensor has to be again inhomogeneous (we could consider magnetic fluxes for gauge fields localized on the brane in order to generate it, for instance).

These 7-branes intersect by pairs on codimension 4 branes with the following energy-momentum tensor,

$$T_{\mu\nu}^{\mathrm{Cod}\,4}(1) = -\eta_{\mu\nu}(1-\beta_2)(1-\beta_3)\left(4\alpha_2 - \frac{24}{R^2}\alpha_3\right)\frac{\delta(\sin\theta_2)}{\sqrt{\kappa_2}}\frac{\delta(\sin\theta_3)}{\sqrt{\kappa_3}}\,, \tag{3.14}$$

$$T_{ij}^{\mathrm{Cod}\,4}(1) = -4\alpha_2\kappa_{ij}(1-\beta_2)(1-\beta_3)\frac{\delta(\sin\theta_2)}{\sqrt{\kappa_2}}\frac{\delta(\sin\theta_3)}{\sqrt{\kappa_3}}\,, \qquad i,j=\theta_1,\phi_1\,, \tag{3.15}$$

where we have shown as an example the intersection at $\sin\theta_2 = \sin\theta_3 = 0$, the other two pairs having a similar energy-momentum tensor.

Finally, and thanks to the third Lovelock term, there is a non-trivial contribution to the generalised Einstein tensor that allows us to match two codimension 6 branes at the intersection of the three codimension 2 branes (i.e. at the points $\theta_1 = \theta_2 = 0$ and $\theta_3 = 0$ or $\theta_3 = \pi$) with the following energy-momentum tensor

$$T_{\mu\nu}^{\mathrm{Cod}\,6} = 24\alpha_3\eta_{\mu\nu}(1-\beta_1)(1-\beta_2)(1-\beta_3)\frac{\delta(\sin\theta_1)}{\sqrt{k_1}}\frac{\delta(\sin\theta_2)}{\sqrt{k_2}}\frac{\delta(\sin\theta_3)}{\sqrt{k_3}}\,. \tag{3.16}$$

Again our general discussion tells us that the fourth Lovelock term contributes to the 3-brane matching condition a term proportional to the Einstein tensor for the induced metric on the brane. In our case this tensor does of course vanish as corresponds to flat space. Notice that the energy-momentum tensors of all the branes are fixed in terms of the deficit angles and R, since we are imposing a particular background. So one should fine tune the brane tensions and the *vevs* of the brane fluxes in order to find this solution.

4. Conclusions

In Einstein gravity there are no regular solutions representing isolated sources of codimension higher than 2. This means that classical solutions of higher dimensional gravity representing branes of codimension three or higher are singular when we model the brane as a delta-like contribution to the energy-momentum tensor. So in order to extract any useful information about the behaviour of gravity on or close to the brane we have to go beyond this approximation (*i.e.* to a theory in which the brane has finite width and some internal structure and/or to a theory that resolves the singularities appearing in classical gravity). In this letter, generalising the ideas presented in [14], we have shown that when we consider all available Lanczos-Lovelock terms in higher dimensions, it is possible to build solutions representing infinitesimally thin branes of higher codimension living in the intersection branes of lower codimension, in a background that is otherwise free of singularities, and in particular where the induced metric in all the branes is well defined. The building blocks of this constructions are codimension 1 and 2 branes, since it is known how to generate isolated one- and two-dimensional delta functions in the Riemann tensor, while the higher dimensional deltas are obtained in the higher order Lovelock terms as the product of the ones appearing in the Riemann tensor. The structure of the Lanczos-Lovelock lagrangians, and in particular the quasilinearity of the equations with respect to the second derivatives of the metric, ensures that singularities of the type $\delta(y)^2$ are not present in the generalised Einstein tensor. Moreover, the delta functions only appear in the components of this tensor along the brane dimensions, and thus this solutions have a natural interpretation as braneworlds.

We have analysed the structure of the matching conditions for the intersection of codimension 1 and/or codimension 2 branes. For the codimension 2 case, under the assumption that the curvature in the bulk does not diverge as we approach the brane, this equations are remarkably simple, since they imply that the induced metric and matter on the brane satisfy the Lovelock equations corresponding with its dimensionality. In particular one obtains 4D Einstein gravity in the case of a 3-brane embedded in a $2n-$dimensional spacetime. We have presented two explicit examples, the intersection of a 5-brane with a 4-brane in 7D, yielding a codimension 3 brane, and the intersection of three 7-branes in 10D, yielding three 5-branes and two 3-branes in its intersections. All of these branes have a non-trivial energy-momentum tensor and its effect in the background is fully taken into account.

Our results open up the possibility of studying cosmology and other gravity related phenomena in ten dimensional intersecting braneworlds within the framework of classical gravity, and provide the first example of non-singular higher codimension braneworlds where the backreaction of the branes in the background is fully taken into account.

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